Revisited water-oriented relationships between a set of farmers and an aquifer: accounting for lag effect

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Abstract

Many environmental problems are due to damages caused by stock of pollutants which accumulate with time lag to their emission. In this paper, we focus on nitrates used in agriculture which can pollute groundwater years after their initial use. A dynamic optimal control problem with heterogeneous farmers is proposed. Usual structural parameters like the discount rate, the natural clearing rate, the lagged time interval between the soil-level pollution occurrence and the impact on groundwater are taken into account. We also examine pollution as caused by a continuous set of farms characterized by their individual performance index and by their individual marginal contribution to the pollution. The issue is further investigated by taking account of change in the information context, successively related to perfect information and to asymmetric information. As a result, when the delay between the spreading of N-fertilizer and the impact on the aquifer increases, i.e., the higher the lag, the steady state pollution stock and the steady state shadow price of the stock both increase. Moreover, asymmetric information leads to a higher stock of pollution. Given the U.S. EPA context or given the European Union context and their directives focusing on nitrate pollution and water quality, the qualitative results provided in this paper should help modellers and decision makers promote suitable environmental policies.

Key-words: non-point source pollution, farming pollution, aquifer, nitrate, time lag, optimal control, mechanism design

Code JEL: Q53, C61, D62, D82
1 Introduction

Aquifers constitute about 89% of the freshwater on our planet, providing most of the drinking water in the world, and they are vulnerable to surface pollution especially from agricultural nitrates (Koundouri 2004). When nitrates are ingested in too large quantities, they have a toxic effect on human health such as baby-blue syndrome and stomach cancer (Addiscott 1996). More generally, the nitrates problem “has the potential to become one of the costliest and the most challenging environmental” the environmental agencies face (Stoner 2011). The U.S. EPA established a maximum contaminant level (MCL) of 10 mg.l⁻¹ for nitrate in drinking water (U.S. EPA, 1995). Moreover, nitrates also contribute to soil eutrophication. Therefore, to preserve the water quality is an important issue. The Water Framework Directive (2000/60/EC) adopted by the European Commissions requires all ground water bodies to achieve a good status by 2015. This goal includes nitrates limit of 50 mg.l⁻¹ set by the Nitrate Directive (91/676/EEC). However, this threshold is already exceeded in many groundwater bodies in Europe (Rivett et al. 2008).

Water pollution problem by nitrates from agriculture is a typical case of nonpoint source (NPS) pollution because individual emissions cannot be measured precisely by the social planner. NPS pollution problems have received considerable attention in the economic literature, mainly to identify the appropriate regulation instruments. These include various mechanism, essentially based on ambient concentration of pollutants (e.g., Segerson 1988, Xepapadeas 1991) and on emission proxies like the inputs (e.g., Griffin and Bromley 1982, Shortle and Dunn 1986, Shortle and Abler 1994). Moreover, managing such pollution is made more difficult by the fact that the social planner is face with a situation of moral hazard and adverse selection. Firstly, in the case ambient-pollutant based instrument, it could be prohibitively costly to measure with sufficient precision the farmers’ actual efforts.
in pollution abatement. Indeed, the social planner can only measure ambient pollutant concentration at prespecified ‘receptor points’. To eliminate this moral hazard problem, Xepapadeas (1991) proposes a system of subsidies and random penalties in case of nonrespect of desired ambient levels, and Bystrom and Bromley (1998) suggest the use of non-individual contracts and collective penalties. Secondly, the social planner faces also an adverse selection problem (both in the case of ambient and input based instrument) which may be related to soil spatial heterogeneity which means that the same management for the same crop in different fields will not necessarily lead to similar nitrate losses (Lacroix et al., 2005; Cabe and Herriges, 1992).

Aquifer polluted by nitrates is similar with a stock pollutant problem which requires to take into account the dynamics of pollutant accumulation in order to regulate efficiently the pollution. However, few studies has considered the dynamic characteristics of this pollutant as noted by Shortle and Horan (2001). Xepapadeas (1992), shows that applying static ambient-incentive schemes in dynamic situations leads to suboptimalities (pollutant overaccumulation), particularly when polluters follow feedback strategies. Then, he proposes schemes which takes the form of charges per unit deviation between desired and observed pollutant accumulation paths. However, due to the slow transfer of nitrates through the unsaturated zone of aquifers, this effects are visible 10-60 years after their use (the nitrate transfer velocity varies between 0.60 and 2.50 m.year$^{-1}$ (Legout et al., 2007; Gutierrez and Baran, 2009)).

Existing lag in nitrate pollution invalidate the incentive schemes proposed by Xepapadeas (1992). Indeed, the social planner cannot impose penalties today when the pollution is due to fertilizers used many decades ago. In this context, optimal regulation of aquifer polluted by nitrates require to take into account the lag effect in a dynamic framework.

In this paper, we examine an optimal management problem of a NPS pollution to a pollutant stock which accumulates with a lag. In the economic literature, the delay between agent’ action or decision and his consequences has been firstly introduced into the accumulation of capital (Rustichini (1989),
Asea and Zak (1999), and more recently into the pollutant accumulation problem (Brandt-Pollmann et al., 2008; Winkler, 2010). They analyze a point source problem in a generic optimal control model devoted to stock accumulation with a fixed delay. We aim to extend this literature to the case of NPS pollution with heterogeneous agents and selection adverse problem. To our knowledge, literature does not deliver how accounting for lag effect in pollution stock can modify the policy of the social planner. A fortiori the problem strengthens in case of asymmetric information. To address this issue, we develop an optimal control problem with heterogeneous agents (farmers) and which emissions accumulate with a time lag. The pollution is caused by a continuous set of producers characterized by their individual performance index, the soil quality, and by their individual marginal contribution to the pollution. Following Winkler (2010) who shows that the a unique saddle point stationary state exists and that the system dynamics crucially depends on the functional form of the objective function, we assume a separable objective function. This allows us to solve the lag problem in the case of perfect and asymmetric information. We show that the lag acts as increasing the stock and its shadow price at the steady state. It implies that the social planner have to impose more efforts to the farmers while the environmental result is weaker. This effect is augmented when asymmetric information occurs between the farmers and the social planner.

The paper is organized as follows. Section 2 is devoted to the presentation of the basic model. In section 3, we set out the generic control problem with time lagged stock accumulation when the social planner is completely informed about individual farm characteristics. In section 4, we develop the analysis of the optimal control problem when asymmetric information drives the mechanism design which should be implemented by the regulator. Finally in section 5, we illustrate differences between accounting for different time lag. We also compare results in case of perfect information to results in case of asymmetric information.
2 Basic elements of the model

Let us consider the set of farmers contributing to the nitrate pollution of an aquifer. Farming activity is represented by the demand of N-fertilizer which is denoted by $x$. Activity depends on performance characteristics summarized by the one-dimensional $\theta$ parameter. The individual farm profit is represented by the function $\pi(x, \theta)$, in which the farm performance characteristics $\theta$ is spread over the interval $\Theta = [\underline{\theta}, \bar{\theta}]$. The likelihood density is denoted by $\gamma(\theta)$ and assumed to be strictly positive at any point within the interval:

$$\gamma(\theta) > 0 \quad \forall \theta \quad (H1)$$

The related cumulative function is denoted by $\Gamma(\theta)$. In agriculture, nitrate losses depend on N-fertilizer demand and use at the soil-root zone. The social planner is assumed to know the aquifer characteristics and the transfer process from arable soils to aquifers at the regional scale by the mean of hydrogeological modelling. Asymmetric information comes with wide heterogeneity of soil and farm characteristics. In other words, when asymmetric information on farm characteristics comes in our analysis, we face an adverse selection problem.

The $\pi$ function is assumed to be twice continuously differentiable. The usual assumption of decreasing return to scale and the assumed positive marginal profit when $x$ is close to 0 hold here:

$$\pi_{xx} < 0 \quad (H2)$$

$$\pi_x(0, \theta) > 0 \quad \forall \theta \quad (H3)$$

We assume that the marginal profit variation regarding the $\theta$ characteristics keeps the same sign. We choose the positive sign, so the marginal profit...
The factor demand increases when the performance parameter increases, and the factor demand decreases when the \( x \)-tax increases. In other words, the N-fertilizer demand increases with the soil quality and decreases with the input-based tax.

The farming activity is assumed to occur over time. At any time \( t \), the \( \theta \)-farm use of \( x \) leads to an increase in the global farming profit by \( \pi(x(\theta, t)), \theta) \) (normalized by prices). Accordingly, the time-unit global profit is expressed by \( \int_{\Theta} \pi(x(\theta, t)), \theta) \gamma(\theta)d\theta \).

Regarding the environmental impact and related damage, we can start by applying a standard framework. The state of our aquifer system is characterized by the nitrate stock per volume unit and denoted by \( z \). The dynamic evolution over time is the result of a double-side effect. On one hand, the clearing effect takes the form of an usual exponential decline characterized by the decline rate \( \tau \). On the other hand, the amount of N-fertilizer consumed by the \( \theta \) farm additively contributes to increase the pollution. The marginal contribution related to \( x \) depends on \( \theta \) and the time-unit contribution of the \( \theta \) farm is \( a(\theta) x(\theta, t) \).

However, a slight difficulty arises when we introduce the lag effect of N-fertilizer use on the nitrate concentration in the aquifer. We denote the lag parameter (not depending on \( \theta \)) by \( \beta \). The time evolution of the environmental
system is described by the equation:

\[
\dot{z}(t) = -\tau z(t) + \int_{\Theta} a(\theta)x(\theta, t - \beta)\gamma(\theta) d\theta
\]  

(1)

Expecting that the regulatory body will be asked to design the optimal individual farm demand for the input \(x(\theta, t)\) at time 0 for any further time \(t\), we assume that the body integrates knowledge related to the initial state of the aquifer and to the short past farming activity. In addition, we recall that the input has to be non-negative. This is expressed by the following assumption:

\[
z(0) = z_0 ; \quad x(\theta, t) = \epsilon(\theta, t) \quad \forall \theta \in \Theta \quad \forall t \in [-\beta, 0] ; \quad x(\theta, t) \geq 0 \quad \forall \theta \in \Theta, \forall t \geq 0
\]  

(H5)

The time unit damage function is expressed by the twice differentiable function depending on \(z\) and denoted by \(D(z)\). The assumptions related to the damage function are:

\[
D_z(0) = 0 \quad \text{and} \quad D_{zz} > 0
\]  

(H6)

Let us notice that [H6] and leads to \(D_z > 0 \forall z > 0\).

Finally, the discount rate is denoted by \(\delta\), and the marginal cost of public funds is denoted by \(\rho\). This last parameter enters the analysis when contractual incentives are taken into consideration.

The economic analysis that follows is based on a partial equilibrium approach with no price feedbacks from the rest of the economy.
3 Long run optimal trade-off between production and pollution in the case of complete information

When information upon farmers is complete, the social planner’s objective is:

\[ W = \int_{0}^{\infty} \left[ \int_{\Theta} \pi(x(\theta, t), \theta) \gamma(\theta) d\theta - D(z(t)) \right] e^{-\delta t} dt \]  \hspace{1cm} (2)

Accordingly, this programme is expressed below:

\[
\max_{x(\theta,t)} W \quad \text{subject to (1), (H5)} \quad (3)
\]

Differently from the usual optimal control programme, the lag term appearing in the state dynamics (1) does not allow us to directly apply the Pontryagin theorem. The solution arises when we consider the transformation of the command variable \( y(\theta, t) = x(\theta, t - \beta) \). The objective function and the state evolution equation are transformed as follows:

\[
W = -\int_{-\beta}^{0} \int_{\Theta} \pi(x(\theta, t), \theta) \gamma(\theta) d\theta e^{-\delta t} dt + \int_{0}^{\infty} \left[ e^{\delta \beta} \int_{\Theta} \pi(y(\theta, t), \theta) \gamma(\theta) d\theta - D(z(t)) \right] e^{-\delta t} dt
\]

\[
\dot{z}(t) = -\tau z(t) + \int_{\Theta} a(\theta) y(\theta, t) \gamma(\theta) d\theta \]

Thanks to the (H5) assumption, the first integral component of this last \( W \) expression can be taken out of the programme. Aiming at the use of the maximum principle, we define the current-value Hamiltonian in which the shadow price of the pollution stock denoted by \( \lambda(t) \) and is designed as to take
a positive value:

\[ H^c = e^{\delta \beta} \int_\Theta \pi(y(\theta, t), \theta)\gamma(\theta)d\theta - D(z(t)) - \lambda(t)[\int_\Theta a(\theta)y(\theta, t)\gamma(\theta)d\theta - \tau z(t)] \]

(6)

According to our technical assumptions, the Pontryagin theorem delivers the conditions holding the optimal solution: \( \{y^*(\theta, t), z^*(t), \lambda^*(t)\} \):

\[ y^*(\theta, t) \text{ maximizes } H^c(y, z^*, \lambda^*) \]  
(7)

\[ \dot{\lambda}^*(t) - \tau \lambda^*(t) = H^c_z(y^*, z^*, \lambda^*) \]  
(8)

Our “convex” problem leads to the following equations:

\[ \pi_x(y^*(\theta, t), \theta) = a(\theta)\lambda^*(t)e^{-\delta \beta} \]  
(9)

\[ \dot{\lambda}^*(t) - (\tau + \delta)\lambda^*(t) = D_z(z^*(t)) \]  
(10)

The transversality condition enters the complete system of conditions:

\[ \lim_{t \to \infty} \lambda(t)e^{-\delta t}z(t) = 0 \]  
(11)

Condition (9) expresses that the \( \theta \) farmer’s profit provided by one additional unit of polluting input equals the discounted cost of the related marginal pollution evaluated at time \( t + \beta \) and weighted by the individual polluting contribution \( a(\theta) \). The \( y \) solution of this equation arises through the relation (R1) \( y^*(\theta, t) = \phi(\theta, a(\theta)\lambda(t)e^{-\delta \beta}) \). The complete solution of the Regulator’s program is provided by the implicit relation between the command \( x \) and the shadow price \( \lambda \), and by the two-dimension differential system, as summarized by the equation set (R2):
∀θ, ∀t > 0 : \( x^*(\theta, t) = \phi(\theta, a(\theta)\lambda(t + \beta)e^{-\delta\beta}) \)

∀θ ∈ Θ, ∀t ∈ [−β, 0] : \( x^*(\theta, t) = \epsilon(\theta, t) \)

\( \dot{z}^*(t) = −τz^*(t) + \int_{\Theta} a(\theta)x^*(\theta, t − \beta)γ(\theta)d\theta \)  \( (R2) \)

\( \dot{\lambda}^*(t) - (\tau + \delta)\lambda^*(t) = −D_z(z^*(t)) \)

\( z^*(0) = z_0 \); the tranversality condition satisfied

There is only one steady state related to this system (proof in appendix A).

The technical assumptions described above lead to deliver too a graphics describing the paths related to this differential system.

Figure 1: Phase diagram describing the paths linking the pollution state \( z \) and its shadow price \( \lambda \).

Let us focus on the steady state \( (\bar{z}, \bar{\lambda}) \) defined by \( \{\dot{y} = 0; \dot{z} = 0\} \). We are interested by the impacts of the parameters \( \beta, \delta, \tau \) on the steady state, leading us to summarize results in propositions 3.1, 3.2 and 3.3.

**Proposition 3.1** When the delay is increased between the spreading of \( N \)—fertilizer on the farm and the impact of it, i.e., the higher the lag, the greater the increase the pollution level and the higher the shadow price in the steady state.
The second next propositions are expected and refer to more usual approaches. It is of interest to keep in mind the main parameter sensitiveness.

**Proposition 3.2** When the discount rate increases, the steady state pollution level and the steady state shadow price both increase.

**Proposition 3.3** When the decline rate increases, i.e more nitrates are absorbed by the aquifer the steady state pollution level and the steady state shadow price both increase.

Proofs are delivered in appendix B.

Having in mind the contractual approach which supports the analysis of the asymmetric information problem (see the section 4), we introduce the Regulator’s choice in supplying contracts to any \( \theta \) farm. A contract is characterized by a two dimensions function \((q(\theta, t), s(\theta, t))\) in which \( q \) refers to the upper limit of \( x \)-use of polluting input and \( s \) refers to the individual fund transfer as the counterpart of profit decrease. Contracts are designed to be freely accepted by the farms, consequently the Regulator has to prevent farmers from refusing the contracts when their participation is expected as socially beneficial.

The transfers call for costly public funds (i.e. one budget unit costs \( 1 + \rho \)) and the social objective is now expressed like:

\[
W = \int_0^{\infty} \{ \int_{\Theta} [\pi(q(\theta, t), \theta) - \rho s(\theta, t)] \gamma(\theta)d\theta - D(z(t)) \} e^{-\delta t} dt \tag{12}
\]

In the complete information case, there is no place for informational rent. The reservation utility of the \( \theta \) farm is the unconstrained profit characterized by the \( q \)-consumption equal to \( \phi(\theta, 0) \) (constant along time). When public funds are costly the individual discounted transfer is equal to the individual profit variation:

\[
\int_0^{\infty} s(\theta, t)e^{-\delta t} dt = \int_0^{\infty} [\pi(\phi(\theta, 0), \theta) - \pi(q(\theta, t), \theta)]e^{-\delta t} dt \tag{13}
\]
The public objective can be rewritten by substitution of the transfer expressed above, so that the Regulator’s programme is now:

$$\max_{q(\cdot, \cdot)} W = \int_0^\infty \{ \int_\Theta [(1 + \rho)\pi(q(\theta, t), \theta) - \rho\pi(\phi(\theta, 0), \theta)]\gamma(\theta)d\theta - D(z(t))\}e^{-\delta t}dt$$

(14)

arising with the unchanged dynamics of the state variable (still delivered by equation [5]). The implicit solution of this programme is still provided through the change in the control variable with respect to the time lag parameter $\beta$.

The contract (for any $\theta$ at any time for the quota $q$, and under an integral equation for any $\theta$-transfer $s$) and the $(z, \lambda)$ path are completely characterized by the system [R3]:

$$\forall \theta, \forall t > 0 : q^*(\theta, t) = \phi(\theta, a(\theta)\lambda^*(t+\beta)e^{-\delta t})$$

$$\forall \theta : \int_0^\infty s^*(\theta, t)e^{-\delta t}dt = \int_0^\infty [\pi(\phi(\theta, 0), \theta) - \pi(q^*(\theta, t), \theta)]e^{-\delta t}dt$$

$$\forall \theta \in \Theta, \forall t \in [-\beta, 0] : q^*(\theta, t) = \epsilon(\theta, t)$$

$$\dot{z}^*(t) = -\tau z^*(t) + \int_\Theta a(\theta)q^*(\theta, t - \beta)\gamma(\theta)d\theta$$

$$\dot{\lambda}^*(t) - (\tau + \delta)\lambda^*(t) = -D_z(z^*(t))$$

$$z^*(0) = z_0 \ ; \ \text{the tranversality condition satisfied}$$

When the parameter related to the shadow cost of public funds tends toward 0 (i.e. $\rho \to 0$), the system [R3] tends toward the system [R2]. The non-costly transfers do not affect the solution $(q, z, \lambda)$.

The parameters $\theta$, $\lambda$ and $\delta$ have similar effects on the steady state as mentioned in the [R2]-analysis. Proposition 3.4 delivers the qualitative impact of the cost of public funds on the steady state (proof in appendix B).

**Proposition 3.4** When the marginal cost of public funds is increased, the greater the increase in pollution level, the higher the shadow price in the steady state.
4 The dynamic problem in the case of asymmetric information

In the case of asymmetric information, the social planner has no individual information on any $\theta$ farm, but he knows the statistical distribution of $\theta$. In others words, the social planner is unable to assess the nitrate losses related to each farm. But she is assumed to know the distribution of the information characteristics. We place our adverse selection problem in the framework of the incentive theory developed by Laffont and Tirole (1993) among others. We consider that the regulator offers a menu of contracts to any farm, and either the farmer $\theta$ selects one of the contracts or he refuses all of them. The problem of the regulator is to design the optimal menu regarding the social objective including the farm profits, the environmental damage, and the regulation costs.

The menu of contracts is a two dimension function $(q(\theta,t), s(\theta,t))$. Like in the previous complete information context, $q$ denotes the “quota” and $s$ denotes the “subsidy”. Formally the regulator acts as asking any farmer at time 0 for contracting or not, and for the characteristics of his $\theta$ farm in the case of acceptance. The participating farmer selects a contract through the announce $\tilde{\theta}$. The acceptance by the farmer implies that he complies at time 0 with the upper bound $q(\tilde{\theta},t)$ holding the $q$-input at any time $t$. He will receive the transfer $s(\tilde{\theta},t)$.

The $\theta$ farmer’s programme is to declare his optimal announce. Based on the revelation principle, the menu proposed by the regulator is a mechanism designed in such a way that the $\theta$ farmer’s dominant strategy is to announce his true characteristics $\theta$. Theoretically the regulator keeps the possibility to design the menu in such a way that the optimal set of participating farmers is a subset of $\Theta$. This opportunity is explored in some papers devoted to application of the incentive theory (see Bourgeon et al. (1995)). For simplicity, we do not keep here this opportunity, even if the menu is possibly suboptimal.

Formally we consider that the functions $q$ and $s$ have the requested math-
ematical properties allowing to use derivatives as long as necessary. The first step of the analysis leads to characterize the incentive constraints and the participation constraint (the so-called rationality constraint). The starting point is the following $\theta$ farmer’s programme which defines the farmer’s optimal announce:

$$\max_{\tilde{\theta}} \int_{0}^{\infty} \left[ \pi(q(\tilde{\theta}, t), \theta) + s(\tilde{\theta}, t) \right] e^{-\delta t} dt$$

(15)

Let us notice that the private discount rate is supposed to be equal to the public one $\delta$. Solving this programme using first and second order conditions, and with the help of the revelation principle, we derive incentives constraints summarized by the relations $\text{IC1}$ and $\text{IC2}$

$$\int_{0}^{\infty} [\pi_x(q, \theta) \frac{\partial q}{\partial \theta} + \frac{\partial s}{\partial \theta}] e^{-\delta t} dt = 0 \quad (\text{IC1})$$

$$\int_{0}^{\infty} \pi_x(q, \theta) \frac{\partial q}{\partial \theta} e^{-\delta t} dt > 0 \quad (\text{IC2})$$

The contract is supposed to be freely accepted by the $\theta$ farmer. When the regulator aims at leading the farmer to accept the contract, he has to ensure that the farmer does not lose with the contract acceptance. The “reservation profit” of the $\theta$ farm is expressed by $\pi(\phi(q,0), \theta)$ (which has a constant current value along time). That leads to define the information rent $R(\theta)$ which has to be not negative as following:

$$R(\theta) = \int_{0}^{\infty} \left[ \pi(q(\theta, t), \theta) + s(q(\theta, t), \theta) - \pi(\phi(q(\theta,0), \theta)) \right] e^{-\delta t} dt \geq 0$$

Assumption $\text{H4}$ leads to demonstrate that the rent decreases when the $\theta$ type increases. Considering that a contract has to be accepted by any $\theta$ in $\Theta$, we can write the rationality constraint under the form $\text{IR1}$ in which only the upper type $\bar{\theta}$ plays:

$$R(\bar{\theta}) = \int_{0}^{\infty} \left[ \pi(q(\bar{\theta}, t), \bar{\theta}) + s(\bar{\theta}, t) - \pi(\phi(q(\bar{\theta},0), \bar{\theta})) \right] e^{-\delta t} dt \geq 0 \quad (\text{IR1})$$
Let us consider the social welfare function $W$ which is now:

$$W = \int_0^\infty \left\{ \int_0^\theta [\pi(q(\theta, t), \theta) - \rho s(q(\theta, t), \theta)] \gamma(\theta) d\theta - D(z(t)) \right\} e^{-\delta t} dt \geq 0$$

The subsidy term is easily replaced with the help of the first order incentive condition (IC1) and integration by parts:

$$\int_0^\infty \int_0^\theta s(q(\theta, t), \theta) \gamma(\theta) d\theta = \int_0^\infty s(\bar{\theta}, t) e^{-\delta t} dt + \int_0^\theta \pi_x(q, t) \frac{\partial q}{\partial \theta} \Gamma(\theta) d\theta e^{-\delta t} dt$$

Like in the previous sections of the paper, regarding the state dynamic equation (1) which calls for the time lag command variable, we choose to replace the command $q(\theta, t)$ by the variable $r(\theta, t) = q(\theta, t - \beta)$ in the function $W$:

$$W = \int_0^\infty \left\{ e^{\delta \beta} \int_0^\theta [\pi(r(\theta, t), \theta) \gamma(\theta) - \rho \pi_x(r(\theta, t), \theta) \frac{\partial r}{\partial \theta}(r(\theta, t), \theta) \Gamma(\theta)] d\theta - D(z(t)) \right\} e^{-\delta t} dt$$

In the second line of this expression, the first negative term related to the $\bar{\theta}$ subsidy weighted by $\rho$ should be as small as possible. The rationality constraint (IR1) leads to design the $\bar{\theta}$ contract in such a way that the rent $R(\bar{\theta})$ is equal to zero. The second term of the second line is explicitly computed thanks to the (H5) hypothesis ($r(\theta, t) = q(\theta, t - \beta) = \epsilon(\theta, t - \beta), \forall \ t \in [0, \beta]$).

The optimal control problem of the regulator can be limited to the first line of the expression above, so that the current hamiltonian function related to the problem is:

$$H^c = e^{\delta \beta} \int_0^\theta [\pi(r, \theta) \gamma - \rho \pi_x(r, \theta) r \theta \Gamma] d\theta - D(z) - \lambda \left( -\tau z + \int_0^\theta a r \gamma d\theta \right)$$

The Pontryagin theorem leads to maximize the $H^c$ function according to the command $r$. Regarding the integral form of the hamiltonian as a function of $r$ and $r \theta$, the problem can be solved by the Euler relation ($\frac{\partial H^c}{\partial r} = \frac{d}{dt} \frac{\partial H^c}{\partial r \theta}$).

Finally, the characterization of the full menu of contracts, the dynamic equations describing the evolution of the state $z$ and the shadow price $\lambda$ is sum-
marized by the system $\text{R4}$. \\

∀θ, ∀t > 0 : \quad q^*(θ, t) = \phi(θ, a(θ)λ^*e^{-(t+\beta)} - \frac{\rho}{1+\rho}π_\theta q^*, θ)\left(\frac{Γ(θ)}{γ(θ)}\right)$ \\

$S(\bar{θ}) = \int_0^∞ s(\bar{θ}, t)e^{-δt}dt = \int_0^∞ [π(\phi(\bar{θ}, 0), \bar{θ}) - π(q(\bar{θ}, t), \bar{θ})]e^{-δt}dt$ \\

∀θ : \quad \int_0^∞ \int_0^∞ \pi_\theta(q(u, t), u)\frac{∂q}{∂θ}(u, t)du e^{-δt}dt \\

∀θ ∈ Θ ∀t ∈ [−β, 0] : \quad q^*(θ, t) = ε(θ, t)$ \hspace{1cm} (R4) \\

$\dot{z}^*(t) = −τz^*(t) + \int_θ a(θ)q^*(θ, t − β)γ(θ)dθ$ \\

$\dot{λ}^*(t) − (τ + δ)λ^*(t) = −Dz(z^*(t))$ \\

$z^*(0) = z_0$ \hspace{0.5cm} ; the transversality condition satisfied \\

We assume that added technical conditions referring to the $\text{IC2}$ conditions hold and allow to consider that the necessary conditions delivered by the system $\text{R4}$ describe the optimal solution. The optimal menu of contracts leads the regulator to design the quota $q$ for any $θ$ at any time $t$. The subsidy appears through an integral condition. \\

Compared to the system $\text{R3}$ the steady state related to the system $\text{R4}$ lets an additional negative term appearing in the expression of the optimal quota ($q^* = \phi(θ, a(θ)λ^*e^{-(t+\beta)} - \frac{\rho}{1+\rho}π_\theta q^*, θ)\left(\frac{Γ(θ)}{γ(θ)}\right)$). This additional term does not allow us to deliver a general result in term of lag effect. The sign of third derivatives enters the conditions which lead to the proposition 4.2. Moreover, this sign plays a crucial role in the comparison between system $\text{R3}$ and system $\text{R4}$ (proposition 4.1). \\

**Proposition 4.1** In case of asymmetric information $\text{R4}$, the level of pollution stock, the shadow price and the total amount of instantaneous polluting input at the steady state are higher than in case of perfect information $\text{R3}$ when the third-derivative, $Π_{xxθ}$, is negative. Otherwise, the effects are ambiguous. \\

Proof is delivered in appendix C. \\

**Proposition 4.2** When the delay between the spreading of $N$–fertilizer on the farm and the impact of it is increased, i.e., the higher the lag, the greater
the increase in the pollution level and the higher the shadow price in the steady state, if the third-derivative $\Pi_{xx\theta}$ is negative. Otherwise, the effects are ambiguous.

Proof is delivered in appendix D.

These results implies that the lag effect would differ in the case of asymmetric information compared to the complete information when the derivative, $\Pi_{xx\theta}$, is not negative for all $\theta$.

5 Discussion and perspective

To open the discussion, in this section we present a numerical application of our analytical approach. Numerical simulations are based on the following added elements, i.e. the specification of the damage function, the specification of the profit function, the specification of the density function, and a set of values for parameters. The damage takes an usual quadratic form:

$$D(z) = \frac{k}{2}z^2, k > 0 \quad (16)$$

The profit function is normalized by prices and takes a form in accordance with usual Nitrogen-yield functions suitable for numerous crops:

$$\Pi(x, \theta) = 1 - e^{-\theta x} - x \text{ with } \theta \in [1, e] \quad (17)$$

The function $1 - e^{-\theta x}$ refers to a yield function based on agronomic observations. We note that the third-derivative $\Pi_{xx\theta}$ is negative. Regarding the input and the output in our analysis, we consider the less performing farm such that $\theta = 1$. The best performing consistent with the hypothesis $H4$ is $e$. The contribution of farmers to a stock of pollution is considered here not $\theta$-dependent ($a(\theta) = a$ for any $\theta$). We assume that the density function follows a uniform distribution. As for the exogenous parameters, the selected values of $a$, $k$ and $\rho$ aims at clearly illustrating the different effects (noting that $\rho$ is in line with previous analytical analyses, for example the value proposed...
by Laffont and Tirole (1993)). The value of the discount rate, \( \delta \), suits the one recommended by regulatory bodies (Lebègue et al. (2005)). According to hydro-geologists, a minimum of 10 to 60 years, depending on the aquifer, is necessary for N-fertilizer to leach into the groundwater (Legout et al., 2007; Gutierrez and Baran, 2009). We set an intermediate value, \( \beta = 30 \) years, by default. It is to be noted that the US Ogallala aquifer (covering 8 States of the U.S. and providing 80% of the drinking water of people living within the aquifer boundary) may enter this category of groundwater, regarding the usual values of transfer velocity and dept of water. Finally, aquifers need up to several decades to eliminate traces of N-fertilizers. We thus deduce the decline rate, \( \tau = 0.02 \). Table 5 resumes the values of parameters.

After optimization and solving in perfect information, we obtain the phase diagram illustrated by figure 2 in line with figure 1, and providing figures in addition.\(^1\)

Figure 3 illustrates the lag effect, when we focus on the steady state and on the optimal path for three values of \( \beta \), including the case \( \beta = 0 \) (i.e. no lag) and the two others lags, respectively \( \beta = 15 \) and \( \beta = 30 \) (years). In our example the introduction of a time lag of 15 years increases the pollution stock by fifty percent in the steady state. A lag time of 30 years would double the pollution stock. Meanwhile the higher is the lag time, significantly the higher is the shadow price of the pollution.

An illustration of the lag influence on the dynamics regarding the pollution stock \( z \) is displayed on figure 4. The lag obviously does not only impact the steady state. The pollution stock goes on increasing during the time interval \( [0, \beta] \). In other words the time lag modifies all the dynamics. 5 illustrates the

\(^1\)All computations and related graphs are obtained by the use of Mathematica-7.
Figure 2: Numerical phase diagram in the case of perfect information describing the paths linking the pollution state $z$ and its shadow price $\lambda$. The dashed curves represent the set of points for which time-derivatives (respectively $z$ and $\lambda$) are equal to 0. The two other curves (green and red) passing through the steady state describe respectively the convergent (green) and divergent (red) paths. The optimal path is the green one.

Fact that an optimal management of pollution which take into account the lag implies more efforts ($\lambda$ higher) for the farmers while the environmental results will be weaker.

Asymmetric information implies a cost on the regulatory body side through the informational rent paid to the farmers. The production allowed each farmer is higher than in the case of perfect information. However, some farmers can do no better than not to produce and therefore they receive a subsidy as a compensation for the income loss. The global effect on pollution stock is ambiguous (see proposition 4.2). Regarding our profit function and its negative third-derivative $\Pi_{xx\theta}$, the level of the pollution stock increases when we move from perfect information toward asymmetric information. The time lag effect is amplified in case of asymmetric information. Figure 6 illustrates both the steady state in perfect and asymmetric information for different values of the time lag and for different values of the opportunity cost of public funds. Even when asymmetric information leads to increase the stock pollution and
the shadow price in the steady state, its impact appears as less important than the time lag impact.
Figure 6: Impacts of the time lag \( \beta \) and of the opportunity cost of public funds \( \rho \) on the steady state, given perfect and asymmetric information: the steady state results of matching the green curve (i.e. the optimal path) and the yellow, purple, orange and blue curves which respectively relate to \( \beta = 0 \) and in the case of perfect and asymmetric information, and \( \beta = 30 \) in the case of perfect and asymmetric information, when \( \rho = 0.3 \) on the left and \( \rho = 0.7 \) on the right.

6 Conclusion

We have developed a dynamic economic framework to assess the impacts of lag time on the NPS optimal management. We analyzed this impact with soil heterogeneity and selection adverse problem. The solution takes form on individual contracts between the social planner and the farmers.

We have showed that that the shadow price and the stock of pollutant at the steady state increase with the lag. This result is important for the design of optimal policy by the social planner. Indeed, an optimal management of pollution which takes into account the lag implies more efforts for the farmers while the environmental results will be weaker. In the case of asymmetric information, only a profit function with a third-derivative positive could lead to an ambiguous results. However, for the standard function (e.g; quadratic, Mitscherlich) used to represent the agricultural activities, the asymmetric information strengthens those obtained in perfect information.

The simulated results give an idea to the amplitude of lag effect. For example, the introduction of a time lag of 15 years increases the pollution stock by fifty percent in the steady state. And a lag time of 30 years would double
the pollution stock. These results show that the lag time leads the social planner to shift from the optimal policy more than he would do when neglecting information asymmetry. The regulation policy, viewed through a menu of contracts in our analysis, is appropriate to deal with the heterogeneity of agents polluting an aquifer and with the diversity of aquifers regarding the lag time. Thanks to the wide range of real lag time, the policy has nevertheless to be adapted to each aquifer. Even in the case of homogenous marginal contribution of pollution (i.e. the $a$ parameter) and in the case of weak opportunity cost of public funds (equivalent to the case of complete information when $\rho = 0$), the lag time is one the major physical drivers of the environmental policy, leading the water body to adapt it to the aquifer and not only to the river basin.

For the further research, added simulations based on more realistic parameters regarding the crop system and the hydrological system should be carried out at the appropriate scale, given the USA and European Union policy contexts and directives focusing on nitrate pollution and water quality.

References


APPENDIX

A  Uniqueness of the steady state (R2)

Let us consider the set of points \( \{(z(t), \lambda(t)); \dot{z} = 0\} \). The relation \( \dot{z} = 0 \) is equivalently verified when \( \tau z = \int_{\Theta} a(\theta)\phi(\theta, a(\theta)\lambda^\gamma(\theta))d\theta \). This relation and the relation \( R1 \) imply that \( z \) decreases when \( \lambda \) increases. The considered set of points is a continuous curve of positive value points meeting the \( z \)-axis and the \( \lambda \)-axis.

Let us consider now the set of points \( \{(z(t), \lambda(t)); \dot{\lambda} = 0\} \), i.e. \((\delta + \tau)\lambda = Dz e^{-\delta \beta}\). The assumption \( [H6] \) immediately leads to an increasing (in \( z \) and \( \lambda \)) curve starting from 0 on the \( \{z, \lambda\} \) plane.

There is only one crossing point belonging to the two previous sets.

For further demonstration, any \( x \) variable at the steady state is denoted by \( \bar{x} \).

B  Propositions 3.1, 3.2, 3.3, 3.4

For propositions 3.1, 3.2 and 3.3 we start from the \( R2 \) system at the steady state:

\[
\begin{align*}
\bar{x}_2(\theta) &= \phi(\theta, a(\theta)\bar{\lambda}_2 e^{-\delta \beta}) \\
\tau \bar{z}_2 &= \int_{\theta} \bar{x}_2(\theta)d\theta \\
(\rho + \delta)\bar{\lambda}_2 &= Dz(\bar{z}_2)
\end{align*}
\]
B.1 Influence of $\beta$

We differentiate the previous system with respect to $\beta$.

\[
\begin{align*}
\frac{\partial \bar{x}_2}{\partial \beta} &= \phi_c(-\delta e^{-\delta \bar{\lambda}_2} a(\theta) e^{-\delta \beta}) \\
\tau \frac{\partial \bar{z}_2}{\partial \beta} &= \int_{\theta} a(\theta) \gamma(\theta) \frac{\partial \bar{x}_2}{\partial \beta} d\theta \\
(\rho + \delta) \frac{\partial \bar{\lambda}_2}{\partial \beta} &= D_{zz}(z_2) \frac{\partial \bar{z}_2}{\partial \beta}
\end{align*}
\]

(a1)

(a2)

(a3)

Solving (a3) for $\frac{\partial \bar{\lambda}_2}{\partial \beta}$ and substituting into (a1), we get:

\[
\frac{\partial \bar{x}_2}{\partial \beta} = \phi_c e^{-\delta \bar{\lambda}_2} (-\delta a(\theta) e^{-\delta \beta} + a(\theta) D_{zz}(z_2) \frac{\partial \bar{z}_2}{\partial \beta})
\]

(a4)

Combining (a2) and (a4), we eliminate the term $\frac{\partial \bar{x}_2}{\partial \beta}$ and thus the system can be written as a single expression which depends only on $\frac{\partial \bar{z}_2}{\partial \beta}$

\[
\tau \frac{\partial \bar{z}_2}{\partial \beta} = \phi_c e^{-\delta \bar{\lambda}_2} \int_{\theta} a(\theta)^2 (-\delta \bar{\lambda}_2 + a(\theta) \frac{D_{zz}(z_2)}{\tau + \delta} \frac{\partial \bar{z}_2}{\partial \beta}) \gamma(\theta) d\theta
\]

(a5)

By rearranging terms (a5) can be written as:

\[
[\tau - e^{-\delta \tau} \frac{D_{zz}(z_2)}{\tau + \delta} \int_{\theta} a(\theta)^2 \gamma(\theta) \phi_c d\theta] \frac{\partial \bar{z}_2}{\beta} = -\delta \bar{\lambda}_2 e^{-\delta \beta} \int_{\theta} a^2(\theta) \gamma(\theta) \phi_c d\theta
\]

Since $\phi_c(\theta, c) < 0$ (thanks to R1), $\frac{\partial \bar{z}_2}{\partial \beta}$ is positive. Consequently, we deduce that:

\[
\frac{\partial \bar{z}_2}{\partial \beta} > 0, \quad \text{therefore} \quad \frac{\partial \bar{\lambda}_2}{\partial \beta} > 0 \text{ thakns to a3 and therefore} \int_{\theta} a(\theta) \gamma(\theta) \frac{\partial \bar{x}_2}{\partial \beta} d\theta > 0
\]

B.2 Influence of $\delta$

The proof is the same as above but the R2-system at the steady state is differentiated with respect to $\delta$:
B.3 Influence of $\tau$

The proof is the same as above but the R2-system at the steady state is differentiated with respect to $\tau$:

\[
\begin{align*}
\frac{\partial \bar{x}_2}{\partial \delta} &= \phi_c a(\theta) e^{-\delta \beta} (-\beta \bar{\lambda}_2 + \frac{\partial \bar{\lambda}_2}{\partial \delta}) \\
\tau \frac{\partial \bar{z}_2}{\partial \delta} &= \int_\theta a(\theta) \gamma(\theta) \frac{\partial \bar{x}_2}{\delta} d\theta \\
- (\tau + \delta) \frac{\partial \bar{\lambda}_2}{\partial \delta} + \bar{\lambda}_2 &= D_{zz}(\bar{z}_2) \frac{\partial \bar{z}_2}{\partial \delta}
\end{align*}
\]

B.4 Influence of $\rho$

We start from the R3-system at the steady state:

\[
\begin{align*}
\bar{q}_3 &= \phi(\theta, \frac{a(\theta) \bar{\lambda}_3 e^{-\delta \beta}}{1 + \rho}) \\
\bar{z}_3 &= \frac{1}{\tau} \int_\theta a(\theta) q_3(\theta) \gamma(\theta) d\theta \\
\bar{\lambda}_3 &= \frac{D_{zz} \bar{z}_3}{\tau + \delta}
\end{align*}
\]

Then, we follow the procedure used above after differentiating of the system with respect to $\rho$:

\[
\begin{align*}
\frac{\partial \bar{q}_3}{\partial \rho} &= \phi_c \left( a(\theta) e^{-\delta \beta} \frac{\partial \bar{\lambda}_3}{\partial \rho} (1 + \rho) + a(\theta) \bar{\lambda}_3 e^{-\delta \beta} \right) \\
\bar{z}_3 + \tau \frac{\partial \bar{z}_3}{\partial \rho} &= \int_\theta a(\theta) \gamma(\theta) \frac{\partial \bar{q}_3}{\partial \rho} d\theta \\
(\tau + \delta) \frac{\partial \bar{\lambda}_3}{\partial \rho} &= D_{zz}(\bar{z}_3) \frac{\partial \bar{z}_3}{\partial \rho}
\end{align*}
\]
And finally, we show that: \( \frac{\partial \bar{z}}{\partial \rho} > 0 ; \frac{\partial \bar{\lambda}}{\partial \rho} > 0 ; \frac{\partial \bar{x}}{\partial \rho} < 0 ; \)

\[ \text{C} \quad \text{Proposition 4.1} \]

We apply the Taylor’s theorem at the first order to the R4-system regarding to the steady state:

\[
\begin{align*}
q_4 - q_3 &= \frac{a(\theta)(\bar{\lambda}_4 - \bar{\lambda}_3) - \rho \frac{\Gamma(\theta)}{\Pi_{xx\theta}}(\bar{\lambda}_4 - \bar{\lambda}_3) \Pi_{xx\theta}(\bar{q}_3, \theta)}{1 + \rho} \phi_c(\theta, a(\theta)\bar{\lambda}_3) \\
\tau(\bar{z}_4 - \bar{z}_3) &= \int_\theta a(\theta)(\bar{q}_4 - \bar{q}_3) \gamma d\theta \\
\bar{x}_4 - \bar{x}_3 &= \frac{1}{\tau + \delta} (\bar{z}_4 - \bar{z}_3) D_{zz}(\bar{z}_3)
\end{align*}
\]

(a6)

We rewrite (a6):

\[
(q_4 - q_3) = \frac{a(\theta)(\bar{\lambda}_4 - \bar{\lambda}_3)}{1 + \rho + \rho \frac{\Gamma(\theta)}{\Pi_{xx\theta}}(\bar{q}_3, \theta) \phi_c(\theta, a(\theta)\bar{\lambda}_3)}
\]

Since \( \phi_c(\theta, c) <= 0 \) (R1) and \( \Pi_{x\theta}(q, \theta) >= 0 \) (H3), if \( \Pi_{xx\theta} < 0 \), then \( \bar{z}_4 - \bar{z}_3 > 0 \) (immediate with a7). As a result, \( \bar{z}_4 - \bar{z}_1 > 0 \Rightarrow \bar{\lambda}_4 - \bar{\lambda}_3 > 0 \) and the global instantaneous pollution is such that \( \int_\theta a(\theta)(\bar{q}_4 - \bar{q}_3) \gamma d\theta > 0 \).

\[ \text{D} \quad \text{Proposition 4.2} \]

We differentiate the R4-system from the steady state with respect to \( \beta \):

\[
\begin{align*}
\frac{\partial \bar{q}_4}{\partial \beta} &= \frac{\phi_c}{1 + \rho} \left[ a(\theta)e^{-\delta(\bar{\lambda}_4 - \bar{\lambda}_3)} - \rho \Pi_{xx\theta} \frac{\partial \bar{q}_4}{\partial \beta} \Gamma(\theta) \right] \\
\frac{\partial \bar{z}_4}{\partial \beta} &= \frac{1}{\tau} \int_\theta a(\theta) \frac{\partial \bar{q}_4}{\partial \beta} \gamma(\theta) d\theta \\
(t + \delta) \frac{\partial \bar{\lambda}_4}{\partial \beta} &= D_{zz}(\bar{z}_4) \frac{\partial \bar{z}_4}{\partial \beta}
\end{align*}
\]

(a10) (a11) (a12)

which after rearranging (a10) and combining (a11) and (a12), we get:
\[
\begin{aligned}
\frac{\partial \tilde{q}_4}{\partial \beta} &= \frac{\phi_c \frac{a(\theta)}{1+\rho} e^{-\delta \lambda_4} (\frac{\partial \lambda_4}{\partial \beta} - \delta \lambda_4)}{1 + \frac{\rho}{1+\rho} \Pi_{xx\theta} \phi_c \gamma(\theta)} \\
(\tau + \delta) \frac{\lambda_4}{\partial \beta} &= \frac{D_{zz}(\tilde{z}_4)}{\tau} \int_{\theta} a(\theta) \frac{\partial \tilde{q}_4}{\partial \beta} \gamma(\theta) \, d\theta
\end{aligned}
\quad (a13, a14)
\]

Combining (a13) and (a14), we deduce:

\[
(\tau + \delta) \frac{\lambda_4}{\partial \beta} = \frac{D_{zz}(\tilde{z}_4)}{\tau} \int_{\theta} a(\theta) \frac{\partial \tilde{q}_4}{\partial \beta} \gamma(\theta) \, d\theta
\]

If \( \Pi_{xx\theta} < 0 \), then we immediately get that \( \frac{\partial \lambda_4}{\partial \beta} > 0 \) and \( \frac{\partial \tilde{z}_4}{\partial \beta} > 0 \). The derivative of total amount of instantaneous polluting input at the steady state is such as \( \frac{\partial \tilde{z}_4}{\partial \beta} = \int_{\theta} a(\theta) \frac{\partial \tilde{q}_4}{\partial \beta} \gamma(\theta) \, d\theta > 0 \).